Exponent Properties

Ms. Draper

Vocabulary

Product of Powers

Power of a Power

Quotient of Powers

Zero + Negative Exponents

coefficient

base

$$5\underbrace{x^4}_{\text{power}}$$

exponent

<u>base</u>: # or variable right before the exponent

<u>exponent</u>: little # to the top right of the base; tells how many times to multiply the base by itself

<u>coefficient</u>: big # to the left of the whole power; numerical factor of a term

---

Vocabulary

Product of Powers

Power of a Power

Quotient of Powers

Zero + Negative Exponents

Multiplying Powers

- same base → add exponents
- multiply coefficients

$$x^m \cdot x^n = x^{m+n}$$

Why it works?

$4x^2 \cdot 6x^3 = 4\underbrace{x \cdot x}_{x^2} \cdot 6\underbrace{x \cdot x \cdot x}_{x^3}$

$4 \cdot 6x^{2+3} = 4 \cdot 6x^5$

$24x^5 = 24x^5$

Examples

invisible 1

① $(3x^3y^2)(-6x^1y^5)$

$(3 \cdot -6)(x^{3+1})(y^{2+5})$

$\boxed{-18x^4y^7}$

② $5ab^3 \cdot 4a^5b^9$

$\boxed{20a^6b^{12}}$

Product of Powers

Power of a Power

Quotient of Powers

Power Rule

Distribute the exponent to EVERY part of the base.

* If the base already has an exponent, MULTIPLY the exponents.

$$(xy^m)^n = x^n x^{m\cdot n} \qquad \left(\frac{x}{y^m}\right)^n = \frac{x^n}{y^{m\cdot n}}$$

Why it works?

$$(2x^2)^3 = (2x^2)(2x^2)(2x^2)$$
$$2^3 x^{2\cdot 3} = 2^3 x^{2+2+2}$$
$$8x^6 = 8(x\cdot x)(x\cdot x)(x\cdot x)$$
$$8x^6 = 8x^6$$

Examples

① $(8h^9)^2 = 8^{1\cdot 2} h^{9\cdot 2} = 8^2 h^{11} = \boxed{64h^{11}}$

② $5(k^4)^5 = 5k^{4\cdot 5} = \boxed{5k^{20}}$

*only distribute to inside ( )

③ $\left(\frac{x}{3}\right)^2 = \frac{x^2}{3^2} = \boxed{\frac{x^2}{9}}$

④ $\left(\frac{8x^2}{2y^3}\right)^3 = \left(\frac{8}{2}\right)^3 \frac{x^{2\cdot 3}}{y^{3\cdot 3}} = 4^3 \frac{x^6}{y^9} = \boxed{\frac{64x^6}{y^9}}$

Power of a Power

Quotient of Powers

Zero + Negative Exponents

Power Rule

Dividing Powers

- same base → subtract exponents
- divide coefficients and reduce fractions

$$\frac{x^m}{x^n} = x^{m-n}$$

Why it works?

$$\frac{4x^5}{8x^3} = \frac{4\cancel{x}\cdot\cancel{x}\cdot\cancel{x}\cdot x\cdot x}{8\cancel{x}\cdot\cancel{x}\cdot\cancel{x}}$$

$$\frac{4}{8}x^{5-3} = \frac{4}{8}\,\frac{x^2}{1}$$

$$\frac{1}{2}x^2 = \frac{1}{2}x^2 = \frac{x^2}{2}$$

Anything divided by itself is 1!

Examples

① $\frac{64a^{100}b^{49}}{8a^{50}b^{9}} = \frac{64}{8}a^{100-50}b^{49-9}$

$\boxed{8a^{50}b^{40}}$

② $\frac{14b^{18}h^{10}}{21b^{7}h^{9}} = \frac{14}{21}b^{18-7}h^{10-9} = \frac{2}{3}b^{11}h^{1}$

$\boxed{\frac{2}{3}b^{11}h}$ invisible 1 on the h!

Quotient of Powers

Zero + Negative Exponents

Power Rule

Dividing Powers

- same base → subtract exponents
- divide coefficients and reduce fractions

$$\frac{x^m}{x^n} = x^{m-n}$$

Why it works?

$$\frac{4x^5}{8x^3} = \frac{4\cancel{x}\cdot\cancel{x}\cdot\cancel{x}\cdot x\cdot x}{8\cancel{x}\cdot\cancel{x}\cdot\cancel{x}}$$

$$\frac{4}{8}x^{5-3} = \frac{4}{8}\,\frac{x^2}{1}$$

$$\frac{1}{2}x^2 = \frac{1}{2}x^2 = \frac{x^2}{2}$$

Anything divided by itself is 1!

Examples

1. $\frac{64a^{100}b^{49}}{8a^{50}b^{9}} = \frac{64}{8}a^{100-50}b^{49-9}$

   $\boxed{8a^{50}b^{40}}$

2. $\frac{14b^{18}h^{10}}{21b^{7}h^{9}} = \frac{14}{21}b^{18-7}h^{10-9} = \frac{2}{3}b^{11}h^{1}$

   $\boxed{\frac{2}{3}b^{11}h}$ invisible 1 on the h!

Quotient of Powers

Zero + Negative Exponents

Exponent of Zero

Anything raised to the power of 0 is 1.

$$x^0 = 1 \qquad \left(\frac{5b^2a}{xy^3}\right)^0 = 1$$

Why it works?

$$\frac{x^4}{x^4} = \frac{\cancel{x}\cdot\cancel{x}\cdot\cancel{x}\cdot\cancel{x}}{\cancel{x}\cdot\cancel{x}\cdot\cancel{x}\cdot\cancel{x}}$$

$$x^{4-4} = 1$$

$$x^0 = 1$$

Anything divided by itself is 1!

Examples

1. $4k^0 = 4\cdot 1 = 4$
2. $\frac{5\cancel{x^3}}{\cancel{x^3}} = 5x^{3-3} = 5x^0 = 5\cdot 1 = 5$
3. $\frac{4h^2g^3}{2h^2g} = 2h^0g^2 = 5g^2$

Negative Exponents

A negative exponent flips the base top ⟷ bottom.
After the flip, the exponent becomes positive.

$$x^{-3} = \frac{x^{-3}}{1} = \frac{1}{x^3} \qquad \frac{1}{x^{-3}} = \frac{x^3}{1} = x^3$$

Why it works?

$$\frac{10d^3}{5d^6} = \frac{10}{5}\,\frac{\cancel{d}\cdot\cancel{d}\cdot\cancel{d}}{\cancel{d}\cdot\cancel{d}\cdot\cancel{d}\cdot d\cdot d\cdot d}$$

$$\frac{10}{5}d^{3-6} = \frac{10}{5}\,\frac{1}{d^3}$$

$$2d^{-3} = \frac{2}{1}\,\frac{1}{d^3}$$

$$\frac{2}{d^3} = \frac{2}{d^3}$$

Anything divided by itself is 1!

Examples

1. $\left(\frac{5x^3y^{-4}}{x^7y}\right)^{-2} = \left(\frac{x^7y}{5x^3y^{-4}}\right)^2$

$$\left(\frac{1}{5}x^{7-4}y^{1--4}\right)^2 = \left(\frac{1}{5}x^3y^5\right)^2$$

$$= \frac{1^2}{5^2}x^{3\cdot 2}y^{5\cdot 2} = \boxed{\frac{1}{5}x^6y^{10}}$$

2. $\frac{4x^3}{x^5}\cdot(2y)^{-2} = 4x^{3-5}\left(\frac{1}{2y}\right)^2$

$$= 4x^{-2}\cdot\frac{1}{2^2y^2} = \frac{4}{x^2}\cdot\frac{1}{4y^2} = \boxed{\frac{1}{x^2y^2}}$$

Zero and Negative Exponents

Put It All Together

$$\left(\frac{12xz^3}{2x^3(y^2)^4z^3}\right)^{-1} = \left(\frac{2x^3(y^2)^4z^3}{12xz^3}\right)^{1} \leftarrow \text{invisible}$$

$$= \frac{2}{12}x^{3-1}y^{2\cdot 4}z^{3-3} = \frac{1}{6}x^2y^8z^0 = \frac{x^2y^8}{6}$$

---

* Note on negatives!

$$-2^4 = -\underbrace{(2)(2)}_{4}\cdot\underbrace{(2)(2)}_{4} = -16 \qquad (-2)^4 = \underbrace{(-2)(-2)}_{4}\cdot\underbrace{(-2)(-2)}_{4} = 16$$

* Exponent only applies to what is inside ( )!